Estimating within-cluster and between-cluster spillover effects in randomized saturation designs

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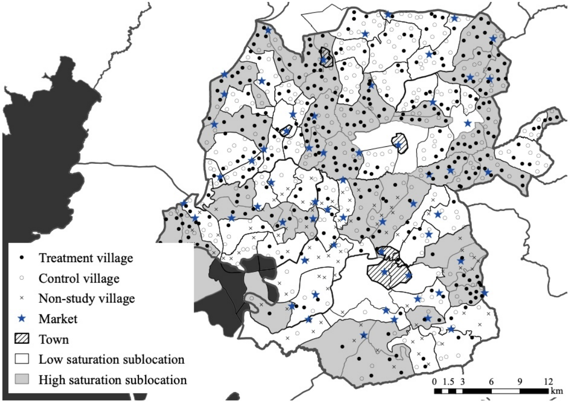
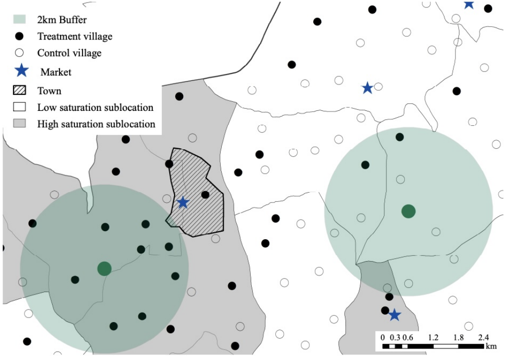
Outline

Motivating example: large-scale cash transfer experiment in rural Kenya (Egger et al., 2022)

- Two-stage *randomized saturation* design:
  - **Stage 1:** Sublocations randomized to high and low saturation.
  - **Stage 2:** Villages randomized to treatment at the assigned saturation rate.
- Geography (markets, labor, mobility) induces interference *within* and *between* sublocations.



Spatial exposure to treatment



- Evidence of meaningful spillovers on non-recipient households and local markets.
- Positive effects on local enterprise activity and consumption reported.

Randomized saturation designs

- Randomize treatment in two stages (Hudgens and Halloran, 2008; Baird et al., 2018; Basse and Feller, 2018):
  1. Stage 1 (cluster level): Randomize clusters (communities, schools, villages, networks) to different saturation levels (high vs. low coverage).
  2. Stage 2 (unit level): Randomize individual units within each cluster to treatment, according to the assigned saturation rate.
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  2. Stage 2 (unit level): Randomize individual units within each cluster to treatment, according to the assigned saturation rate.
- Enables identification of both direct and spillover effects.
- Widely-used in public health and social sciences: deworming on health (Miguel and Kremer, 2004), vaccination on health (Ali et al., 2005), labor market policies on displacement (Crépon et al., 2013), school governance on learning (Pradhan et al., 2014), microcredit on business outcomes (Banerjee et al., 2015), education on fertility (Duflo et al., 2015), cash transfers on school attendance (Barrera-Osorio et al., 2019).

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- Not reasonable in many cases:
  - Real-world clusters are rarely isolated.
  - Networks, mobility, markets, or environmental channels can connect units across cluster boundaries.
  - Ignoring between-cluster interference can bias effect estimates and mislead policy conclusions.

Our contribution and roadmap

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- **Inference**: closed-form asymptotic variance and covariance, joint CLT under network dependence, conservative variance estimators, and Wald CIs.
- **Empirical re-analysis** of Egger et al. (2022): large-scale cash transfer experiment in rural Kenya. Geographic spillovers are economically meaningful and effects are highly heterogeneous across exposure environments.

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- Without further structure, 2^n potential outcomes per unit.

Exposure mapping

- Adopt an exposure mapping to reduce dimensionality:

$$d_i(\boldsymbol{A}) = (A_i, S_i, H_i),$$

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  - H_i: between-cluster summary, e.g., whether $> 1/2$ villages are treated among those geographically close to i but located in different sublocations.
- Potential outcome reduces to $Y_i(a, s, h)$.

Outline

Conditional direct and indirect effects

- Conditional direct effect of A_i, holding (S_i, H_i) fixed at (s, h):

$$\mathrm{DE}(s, h) = n^{-1} \sum_{i=1}^{n} Y_i(1, s, h) - n^{-1} \sum_{i=1}^{n} Y_i(0, s, h).$$

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- Within-cluster conditional indirect effect of S_i, holding $H_i = h$:

$$\text{WIE}(h) = n^{-1}\sum_{i=1}^{n} Y_i(0,1,h) - n^{-1}\sum_{i=1}^{n} Y_i(0,0,h).$$

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- Between-cluster conditional indirect effect, holding $S_i = s$:

$$\text{BIE}(s) = n^{-1} \sum_{i=1}^{n} Y_i(0, s, 1) - n^{-1} \sum_{i=1}^{n} Y_i(0, s, 0).$$

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- All depend on the design through $\mathrm{pr}(S_i, H_i \mid A_i, \cdots)$.

Policy-specific direct and indirect effects

- For a given treatment policy ψ, define

$$\mathrm{DE}_\psi = n^{-1}\sum_{i=1}^{n} E_\psi\{Y_i(1, S_i, H_i) \mid A_i = 1\} - n^{-1}\sum_{i=1}^{n} E_\psi\{Y_i(0, S_i, H_i) \mid A_i = 0\},$$

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  - subscript ψ: $E_\psi(\cdot \mid \cdot)$ is taken under the conditional distribution induced by ψ.
- Comparison between different policies ψ_1 and ψ_2, e.g., $\mathrm{DE}_{\psi_1} - \mathrm{DE}_{\psi_2}$.

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  - Horvitz–Thompson estimator:

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- Conditional effects: differences of $\hat{Y}^*(a,s,h)$
  - e.g., $\hat{\mathrm{DE}}^*(s,h) = \hat{Y}^*(1,s,h) - \hat{Y}^*(0,s,h)$ for $* \in \{\mathrm{ht}, \mathrm{haj}\}$.

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- Practical solution: simulate via Monte Carlo from the known design (Aronow and Samii, 2017).
  - Draw B assignment vectors $\boldsymbol{A}^{(b)}$ following the two-stage design.
  - Compute $S_i^{(b)}, H_i^{(b)}$ on the network for each draw.
  - Estimate $\hat{\pi}_i(a, s, h) = B^{-1} \sum_b \mathbb{1}\{A_i^{(b)} = a, S_i^{(b)} = s, H_i^{(b)} = h\}$.

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- With $\gamma_i \equiv 1$, recover the conditional-effect estimators.

Choice of weights

- Use policy-induced conditional probabilities as weights:

$$\gamma_{i,\psi}^{\mathrm{DE}}(a, s, h) = \mathrm{pr}_{\psi}(S_i = s, H_i = h \mid A_i = a),$$
$$\gamma_{i,\psi}^{\mathrm{WIE}}(a, s, h) = \mathrm{pr}_{\psi}(H_i = h \mid A_i = a, S_i = s),$$
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- Computed via the same Monte Carlo simulation from the design.
- Can be used to construct estimators for policy-specific effects: for $* \in \{\mathrm{ht}, \mathrm{haj}\}$,

$$\hat{\mathrm{DE}}_\psi^* = \sum_{s,h=0,1} \{\hat{Y}^*(1,s,h;\Gamma_\psi^{\mathrm{DE}}) - \hat{Y}^*(0,s,h;\Gamma_\psi^{\mathrm{DE}})\},$$
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Choice of weights

- Use policy-induced conditional probabilities as weights:

$$\gamma_{i,\psi}^{\mathrm{DE}}(a,s,h) = \mathrm{pr}_\psi(S_i = s, H_i = h \mid A_i = a),$$
$$\gamma_{i,\psi}^{\mathrm{WIE}}(a,s,h) = \mathrm{pr}_\psi(H_i = h \mid A_i = a, S_i = s),$$
$$\gamma_{i,\psi}^{\mathrm{BIE}}(a,s,h) = \mathrm{pr}_\psi(S_i = s \mid A_i = a, H_i = h).$$

- Computed via the same Monte Carlo simulation from the design.
- Can be used to construct estimators for policy-specific effects: for $* \in \{\mathrm{ht}, \mathrm{haj}\}$,

$$\hat{\mathrm{DE}}_\psi^* = \sum_{s,h=0,1} \left\{ \hat{Y}^*(1,s,h;\Gamma_\psi^{\mathrm{DE}}) - \hat{Y}^*(0,s,h;\Gamma_\psi^{\mathrm{DE}}) \right\},$$
$$\hat{\mathrm{WIE}}_\psi^* = \sum_{h=0,1} \left\{ \hat{Y}^*(0,1,h;\Gamma_\psi^{\mathrm{WIE}}) - \hat{Y}^*(0,0,h;\Gamma_\psi^{\mathrm{WIE}}) \right\},$$
$$\hat{\mathrm{BIE}}_\psi^* = \sum_{s=0,1} \left\{ \hat{Y}^*(0,s,1;\Gamma_\psi^{\mathrm{BIE}}) - \hat{Y}^*(0,s,0;\Gamma_\psi^{\mathrm{BIE}}) \right\}.$$

- In-policy marginal effects: a special case with the implemented policy.

Outline

Regularity conditions

- Bounded potential outcomes: $|Y_i(a, s, h)| \leq C_Y$ for all i and (a, s, h).

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- Bounded network degree: $\max_i |\{j : k_j = k_i\} \cup \mathcal{G}_i| \leq \Delta < \infty$.
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- Bounded order of dependence: there exists $m < \infty$ such that $(A_i, S_i, H_i) \perp\!\!\!\perp (A_j, S_j, H_j)$ whenever i, j are graph distance $> m$.
  - Dependence does not propagate indefinitely; key for graph-dependent CLT.
  - In the application: $m = 2$.

Asymptotic variance: Horvitz–Thompson

- For the Horvitz–Thompson estimator:

$$\mathrm{var}\{\hat{Y}^{\mathrm{ht}}(a,s,h;\Gamma)\} = \underbrace{n^{-2}\sum_{i=1}^{n}\frac{1-\pi_i(a,s,h)}{\pi_i(a,s,h)}\{\gamma_i(a,s,h)Y_i(a,s,h)\}^2}_{\text{contribution from the variation of unit } i}$$

$$+\underbrace{n^{-2}\sum_{i=1}^{n}\sum_{j\neq i}\frac{\pi_{ij}(a,s,h;a,s,h)-\pi_i(a,s,h)\pi_j(a,s,h)}{\pi_i(a,s,h)\pi_j(a,s,h)} \times \gamma_i(a,s,h)\gamma_j(a,s,h)Y_i(a,s,h)Y_j(a,s,h).}_{\text{contribution from the covariance between units due to interference}}$$

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- Compact quadratic form:

$$\mathrm{var}\{\hat{Y}^{\mathrm{ht}}(a,s,h;\Gamma)\} = n^{-2}\boldsymbol{Y}(a,s,h)^{\mathrm{T}}\boldsymbol{\Gamma}(a,s,h)\boldsymbol{\Lambda}(a,s,h)\boldsymbol{\Gamma}(a,s,h)\boldsymbol{Y}(a,s,h).$$

Asymptotic variance: Hájek estimator

- For the Hájek estimator: same form, but with **centered** potential outcomes

$$Y_i^{\text{haj}}(a,s,h) = Y_i(a,s,h) - \frac{n^{-1}\sum_{i=1}^{n}\gamma_i(a,s,h)Y_i(a,s,h)}{n^{-1}\sum_{i=1}^{n}\gamma_i(a,s,h)}.$$

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- Compactly:

$$\text{avar}\{\hat{Y}^{\text{haj}}(a, s, h; \Gamma)\} = n^{-2} \boldsymbol{Y}^{\text{haj}}(a, s, h)^{\mathrm{T}} \boldsymbol{\Gamma}(a, s, h) \boldsymbol{\Lambda}(a, s, h) \boldsymbol{\Gamma}(a, s, h) \boldsymbol{Y}^{\text{haj}}(a, s, h).$$

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- The centering reduces the magnitude of the quadratic form ⇒ explains why Hájek is usually *more stable* in finite samples.

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&\operatorname{cov}\{\hat{Y}^{\text{ht}}(a, s, h; \Gamma), \hat{Y}^{\text{ht}}(a', s', h'; \Gamma)\} \\
&= \underbrace{-n^{-2} \sum_{i=1}^{n} \gamma_i(a, s, h)\gamma_i(a', s', h') Y_i(a, s, h) Y_i(a', s', h')}_{\text{covariance of different potential outcomes for the same unit } i} \\
&\quad \underbrace{+ n^{-2} \sum_{i=1}^{n} \sum_{j \neq i} \frac{\pi_{ij}(a, s, h; a', s', h') - \pi_i(a, s, h)\pi_j(a', s', h')}{\pi_i(a, s, h)\pi_j(a', s', h')} \times \gamma_i(a, s, h)\gamma_j(a', s', h') Y_i(a, s, h) Y_j(a', s', h')}_{\text{covariance of different potential outcomes between units}}
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- Hájek covariance: Same form but with centered potential outcomes $Y_i^{\text{haj}}(a, s, h)$ and $Y_j^{\text{haj}}(a', s', h')$.

Joint consistency and CLT

Theorem (Consistency & asymptotic normality). Under the regularity conditions and non-degeneracy of the asymptotic variance, for $* \in \{\text{ht}, \text{haj}\}$:

1. Consistency: $\hat{\boldsymbol{Y}}_\Gamma^* - \bar{\boldsymbol{Y}}_\Gamma \xrightarrow{p} 0$.
2. Joint asymptotic normality:

$$\text{acov}(\hat{\boldsymbol{Y}}_\Gamma^*)^{-1/2}\left(\hat{\boldsymbol{Y}}_\Gamma^* - \bar{\boldsymbol{Y}}_\Gamma\right) \xrightarrow{d} \mathcal{N}(0, I_8),$$

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- Proof uses the graph-dependent CLT of Chen and Shao (2004).
- All proposed estimators (conditional, in-policy marginal, policy-specific) are consistent and asymptotically normal by the continuous mapping theorem and the delta method.

Conservative variance estimator

- Define $\boldsymbol{\Omega}(a, s, h)$:

$$\boldsymbol{\Omega}_{ij}(a, s, h) = \frac{\pi_{ij}(a, s, h; a, s, h) - \pi_i(a, s, h)\pi_j(a, s, h)}{\pi_{ij}(a, s, h; a, s, h)},$$

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$$\hat{\text{se}}^2\{\hat{Y}^*(a, s, h; \Gamma)\} = n^{-2}(\hat{\boldsymbol{Y}}^*)^{\mathrm{T}}\boldsymbol{\Gamma}(a, s, h)\boldsymbol{\Omega}(a, s, h)\boldsymbol{\Gamma}(a, s, h)\hat{\boldsymbol{Y}}^*,$$

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- Cross-level covariance is unidentifiable for *contrasts*.
  - Use Cauchy–Schwarz to obtain a **conservative** bound:

$$\hat{\text{var}}\{\widehat{\mathrm{DE}}^*(s, h)\} = \left[\,\hat{\text{se}}\{\hat{Y}^*(1, s, h)\} + \hat{\text{se}}\{\hat{Y}^*(0, s, h)\}\,\right]^2.$$

Asymptotically valid Wald confidence intervals

Corollary. Under the regularity conditions, positivity of second-order inclusion probabilities $\pi_{ij}(a, s, h; a, s, h) > 0$, and non-degeneracy:

1. For each (a, s, h) and $* \in \{\mathrm{ht}, \mathrm{haj}\}$,

$$n\Big[\widehat{\mathrm{var}}\{\hat{Y}^*(a, s, h; \Gamma)\} - \mathrm{avar}\{\hat{Y}^*(a, s, h; \Gamma)\}\Big] = o_p(1),$$

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2. For **conditional** and **policy-specific** effects, the Cauchy–Schwarz-based variance estimator yields CIs with asymptotic coverage *at least* $1 - \alpha$.

- If $\pi_{ij}(a, s, h; a, s, h) = 0$ for some pair, single-cell variance is also not consistently estimable; can use the Aronow and Samii (2017) construction.

Outline

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- In spirit of Lin's interacted regression; rigorous analysis of efficiency under interference is ongoing work.

Outline

Re-analysis of Egger et al. (2022)

- $n = 653$ villages, 155 sublocations, two counties in rural Kenya, 2014–2017.
  - 328 treated, 325 control villages.
  - High-saturation sublocations: $2/3$ villages treated; low-saturation: $1/3$.

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- Two estimators reported: Hájek, covariate-adjusted.

Estimated propensity scores

Exposure	Mean	Std	Median
$(A_i = 0, S_i = 0, H_i = 0)$	0.184	0.175	0.137
$(A_i = 0, S_i = 0, H_i = 1)$	0.143	0.141	0.099
$(A_i = 0, S_i = 1, H_i = 0)$	0.088	0.062	0.071
$(A_i = 0, S_i = 1, H_i = 1)$	0.086	0.068	0.068
$(A_i = 1, S_i = 0, H_i = 0)$	0.118	0.078	0.088
$(A_i = 1, S_i = 0, H_i = 1)$	0.098	0.065	0.078
$(A_i = 1, S_i = 1, H_i = 0)$	0.139	0.143	0.069
$(A_i = 1, S_i = 1, H_i = 1)$	0.144	0.154	0.051
Marginal probabilities			
$\text{pr}(A_i = 1)$	0.499	0.167	0.500
$\text{pr}(S_i = 1)$	0.458	0.342	0.500
$\text{pr}(H_i = 1)$	0.471	0.224	0.500

- Lower-tail of joint propensities is small; positivity is plausible but tight in some cells.

Conditional direct effects $\hat{\text{DE}}^*(s, h)$

(s, h)	Hájek Profit	Hájek Revenue	Hájek Costs	Hájek Wage	Covariate-adjusted Profit	Covariate-adjusted Revenue	Covariate-adjusted Costs	Covariate-adjusted Wage
(0, 0)	−360	−46	200	163	−101	217	172	136
	(909)	(1,143)	(135)	(116)	(883)	(1,117)	(137)	(117)
(0, 1)	−558	300	329**	268*	−835	−124	260*	211
	(616)	(877)	(168)	(145)	(559)	(696)	(148)	(131)
(1, 0)	**3,539*** **	**4,159*** **	87	23	**2,817** **	**3,043*** **	−150	−181
	(1,263)	(1,247)	(169)	(159)	(1,268)	(1,167)	(157)	(149)
(1, 1)	−227	140	115	73	37	481	142	95
	(688)	(825)	(158)	(134)	(677)	(776)	(153)	(131)

- Large positive direct effect on profits/revenue at $(S, H) = (1, 0)$: high local saturation, low geographic exposure.
- Cost/wage effects are positive at $(0, 1)$: competition for inputs/labor when nearby villages are treated.

Conditional within-cluster indirect effects $\widehat{\text{WIE}}^*(a, h)$

(a, h)	Hájek Profit	Hájek Revenue	Hájek Costs	Hájek Wage	Covariate-adjusted Profit	Covariate-adjusted Revenue	Covariate-adjusted Costs	Covariate-adjusted Wage
$(0, 0)$	-182	-304	132	134	67	363	333^{**}	304^{***}
	(837)	(961)	(170)	(148)	(823)	(943)	(159)	(140)
$(0, 1)$	$-1{,}136^{*}$	$-1{,}579^{*}$	-92	-67	$-1{,}333^{**}$	$-1{,}728^{**}$	-76	-50
	(671)	(813)	(138)	(121)	(654)	(750)	(132)	(116)
$(1, 0)$	$\mathbf{3{,}718}^{***}$	$\mathbf{3{,}902}^{***}$	18	-6	$\mathbf{2{,}985}^{**}$	$\mathbf{3{,}189}^{**}$	11	-13
	(1,335)	(1,429)	(134)	(127)	(1,327)	(1,342)	(134)	(126)
$(1, 1)$	-805	$-1{,}738^{**}$	-306	-262^{*}	-461	$-1{,}124$	-194	-166
	(633)	(889)	(188)	(158)	(581)	(722)	(169)	(147)

- Treated villages with low geographic exposure benefit substantially from high local saturation.
- Among control villages with high H, increased local saturation *reduces* profits and revenues (competitive pressure from treated neighbors).

Conditional between-cluster indirect effects $\widehat{\mathrm{BIE}}^*(a, s)$

(a, s)	Hájek Profit	Hájek Revenue	Hájek Costs	Hájek Wage	Covariate-adjusted Profit	Covariate-adjusted Revenue	Covariate-adjusted Costs	Covariate-adjusted Wage
(0, 0)	**1,330****	**1,950*****	222**	176**	**1,442*****	**1,998*****	204**	161**
	(554)	(645)	(93)	(83)	(511)	(555)	(86)	(76)
(0, 1)	375	674	−2	−25	42	−92	−204	−194
	(954)	(1,128)	(214)	(186)	(967)	(1,138)	(205)	(180)
(1, 0)	1,132	2,296*	350*	282	708	1,658	293*	236
	(971)	(1,374)	(209)	(178)	(930)	(1,258)	(199)	(172)
(1, 1)	**−3,391*****	**−3,344*****	26	25	**−2,739*****	**−2,654*****	88	83
	(997)	(944)	(112)	(106)	(978)	(805)	(104)	(101)

- Control villages in low-saturation sublocations benefit from proximity to treated villages, possibly due to increased economic activity.
- Treated villages in high-saturation sublocations are negatively affected by additional geographic exposure: negative competitive spillover.

In-policy marginal effects

	Hájek				Covariate-adjusted			
	Profit	Revenue	Costs	Wage	Profit	Revenue	Costs	Wage
$\hat{\text{DE}}$	336	811	164	121	289	656	99	64
	(1,052)	(1,242)	(232)	(205)	(995)	(1,119)	(215)	(191)
$\hat{\text{WIE}}$	−763	−1,146	53	74	−689	−835	142	148
	(710)	(855)	(168)	(149)	(686)	(818)	(155)	(137)
$\hat{\text{BIE}}$	358	601	75	61	436	565	1	−6
	(1,138)	(1,321)	(217)	(190)	(1,060)	(1,160)	(206)	(181)

- No marginal effect is statistically significant: averaging *dilutes* the highly heterogeneous conditional signals.

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  - Closed-form asymptotic variance and covariance, joint CLT under graph dependence, conservative variance estimators, and Wald CIs.

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- **Empirical lessons** from the cash transfer experiment:
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  - Marginal effects can mask important heterogeneity.
- **Future directions**: continuous / multi-level exposures, formal model-assisted covariate adjustment under interference and regression analysis for randomized saturation designs.

Thank you very much!

Comments and questions welcome.
https://arxiv.org/abs/2603.19573

References I

Ali, M., Emch, M., von Seidlein, L., Yunus, M., Sack, D. A., Rao, M., Holmgren, J., and Clemens, J. D. (2005). Herd immunity conferred by killed oral cholera vaccines in bangladesh: a reanalysis. *The Lancet*, 366(9479):44–49.

Aronow, P. M. and Samii, C. (2017). Estimating average causal effects under general interference, with application to a social network experiment. *The Annals of Applied Statistics*, 11(4):1912–1947.

Baird, S., Bohren, J. A., McIntosh, C., and Özler, B. (2018). Optimal design of experiments in the presence of interference. *Review of Economics and Statistics*, 100(5):844–860.

Banerjee, A., Duflo, E., Glennerster, R., and Kinnan, C. (2015). The miracle of microfinance? evidence from a randomized evaluation. *American Economic Journal: Applied Economics*, 7(1):22–53.

Barrera-Osorio, F., Linden, L. L., and Saavedra, J. E. (2019). Long-term impacts of alternative approaches to increase schooling: Evidence from a randomized experiment in colombia. *American Economic Journal: Applied Economics*, 11(3):33–54.

Basse, G. and Feller, A. (2018). Analyzing two-stage experiments in the presence of interference. *Journal of the American Statistical Association*, 113(521):41–55.

Chen, L. H. and Shao, Q.-M. (2004). Normal approximation under local dependence. *The Annals of Probability*, 32(3A):1985–2028.

References II

Crépon, B., Duflo, E., Gurgand, M., Rathelot, R., and Zamora, P. (2013). Do labor market policies have displacement effects? evidence from a clustered randomized experiment. *Quarterly Journal of Economics*, 128(2):531–580.

Duflo, E., Dupas, P., and Kremer, M. (2015). Education, hiv, and early fertility: Experimental evidence from kenya. *American Economic Review*, 105(9):2757–2797.

Egger, D., Haushofer, J., Miguel, E., Niehaus, P., and Walker, M. (2022). General equilibrium effects of cash transfers: experimental evidence from kenya. *Econometrica*, 90(6):2603–2643.

Hudgens, M. G. and Halloran, M. E. (2008). Toward causal inference with interference. *Journal of the American Statistical Association*, 103(482):832–842.

Miguel, E. and Kremer, M. (2004). Worms: identifying impacts on education and health in the presence of treatment externalities. *Econometrica*, 72(1):159–217.

Pradhan, M., Suryadarma, D., Beatty, A., Wong, M., Alishjabana, A., Gaduh, A., and Artha, R. (2014). Improving educational quality through enhancing community participation: Results from a randomized field experiment in indonesia. *American Economic Journal: Applied Economics*, 6(2):105–126.

What if we ignored between-cluster interference?

Reduced exposure mapping (A_i, S_i) (4 cells), Hájek estimator:

	Profit	Revenue	Costs	Wage
$\widehat{\mathrm{DE}}(s=0)$	-298	147	252^{**}	214^{**}
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- Geographic spillovers absorbed into noise; risk of attenuation and misinterpretation.